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ANALYTICAL SOLUTION OF A PROBLEM OF A COLLISION OF TWO HYPERSONIC GAS FLOWS FROM SYMMETRIC SOURCES

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An asymptotic solution of the Euler equations that describe stationary interaction of two hypersonic gas flows from two identical spherically symmetric sources and an integral equation determining the shock wave shape are obtained with the use of a modified method of expansion of the sought functions with respect to a small parameter, which is the ratio of gas densities in the incoming flow and behind the shock wave. The solution of this equation near the axis of symmetry allows the shock wave stand-off distance from the contact plane and the radius of its curvature to be found. It is shown that the solution obtained agrees well with the known numerical solutions.

Key words: *hypersonic flow, spherically symmetric source, gas-dynamic equations, asymptotic solution.*

Introduction. The problem of incidence of a nonuniform supersonic flow onto a plane target is of interest in studying the dynamic and thermal effect of plasma jets on elements of flying vehicles and on the Earth's surface [1]. It is known that the flow in a supersonic jet escaping from an axisymmetric nozzle into vacuum can be modeled by a flow at a certain distance from a spherical source [2]. A similar problem is encountered in astrophysics in studying a collision of two identical star winds emanating from double star systems [3–5].

If the characteristics of both supersonic sources are identical, then the problem of the collision of two flows reduces to the problem of interaction of a spherically symmetric supersonic flow with a plane target. In accordance with this model, the flows of two gases are divided by a contact surface, which is a plane, and the interaction region is enclosed between two shock waves. Each of the two symmetric domains between the shock wave (SW) and the contact surface consists of subdomains of subsonic, transonic, and supersonic flows. This flow pattern is similar to the pattern of supersonic flow around blunted bodies [6]. Various methods are currently used to calculate stationary problems of an inviscid gas flow around blunted bodies; the most resource-intensive method is based on the time-stabilization principle [1, 2]. Iterative methods, in particular, a method of global iterations, are less resource-intensive [7, 8]. A method of global iterations based on iterations in terms of the bow SW shape and the streamwise pressure gradient in the subsonic flow region was proposed in [7] to solve the problem of a supersonic inviscid gas flow around blunted bodies. A more effective iterative-marching numerical algorithm based on iterations in terms of the degree of deviation of the shape of normalized pressure profiles from the shape of local similarity in the subsonic part of the shock layer was developed in [8]. In contrast to [7], the SW shape in [8] is not specified at each iteration, but is calculated together with other sought functions by the marching method. Though numerical methods for solving gas-dynamic equations have been well developed [9, 10], analytical and approximate solutions of hypersonic gas-dynamic problems are still urgent; they are used: (i) for testing and verifying the convergence of methods of the numerical solution of gas-dynamic equations; (ii) for approximating observation results; (iii) as the initial approximation in iterative methods [6], in particular, in the method of global iterations [7]; (iv) as a basis for solving more complicated problems. The solution of complicated problems with allowance

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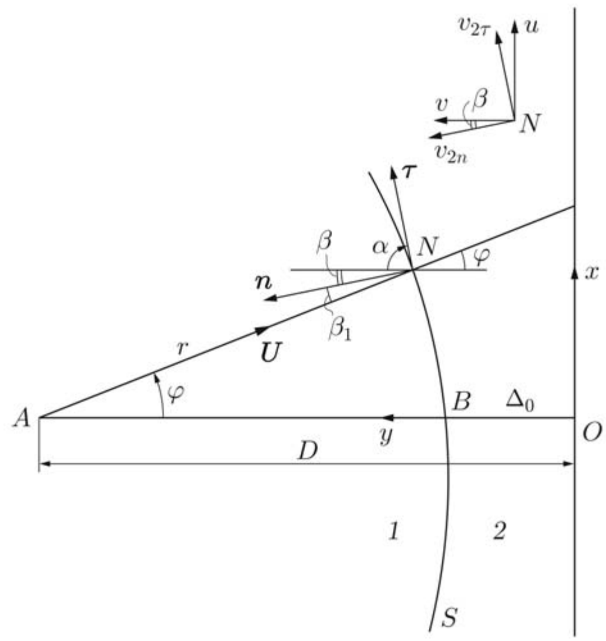

Fig. 1. Schematic of the flow: 1) domain of the flow from one source with the center located at the point A; 2) domain of the flow behind the SW.

for dissipation, radiation, and nonequilibrium processes can be substantially simplified by means of splitting in terms of physicochemical processes [9] and approximate solution of the gas-dynamic part of the problem. One of the most effective approximate methods of solving the stationary Euler equations for a supersonic flow around blunted bodies is Chernyi's asymptotic method [11], which includes expansion of the sought functions with respect to a small parameter, namely, the ratio of gas densities in the incoming flow and behind the SW. This method was widely used to solve problems of a hypersonic uniform flow of a perfect gas around blunted bodies [6, 11] and also problems of nonuniform flow [12] and radiative gas flow [13]. It seems of interest to develop this method for studying a nonuniform gas flow past a plane. Gas-dynamic equations that describe a stationary hypersonic gas flow escaping from a spherical source past a plane were solved numerically (by a time-dependent method) in [1, 2], and an asymptotic solution of this problem was found in [14].

In this work, as in [14], expressions with accuracy to an arbitrary angle of SW inclination were obtained by the method of expansion of the sought functions for gas-dynamic parameters in the shock layer. Based on the parameters found, an integral equation determining the SW shape was derived. The solution of this equation near the axis of symmetry made it possible to find the SW stand-off distance from the contact plane and the radius of its curvature. It was demonstrated that the solution obtained agrees with the known numerical solution [2] much better than the solution obtained in [14].

1. Formulation of the Problem. Let us study a collision of two hypersonic gas flows from identical spherical sources. The distance between the centers of these sources is $2D$. By virtue of symmetry, the contact surface is a plane located at a distance D from the center of each source. Figure 1 shows the flow pattern in the case of a collision of two identical supersonic flows. Let us introduce a cylindrical coordinate system (x, y) fitted to the contact plane. The x distance is counted from the axis of symmetry along the plane, and the y distance is counted along the normal to the plane (positive values of y correspond to the direction toward the source center A). We also introduce a spherical coordinate system (r, φ) with the center at the point A. The angle between the tangential line to the curvilinear shock wave S and the Oy axis is denoted by α. The vectors $\boldsymbol{n}$ and $\boldsymbol{\tau}$ indicate the direction of the tangential line and the normal at an arbitrary point N on the SW. The components of the velocity vector $\boldsymbol{U}$ are v_{1n} and $v_{1\tau}$ ahead of the SW and v_{2n} and $v_{2\tau}$ behind the SW; the velocity components in the coordinate system (x, y) are u and v. The relation of the components of the velocity vector $\boldsymbol{U}$ behind the SW $v_{2\tau}$, v_{2n} with the components u and v of the same vector in the coordinate system (x, y) is found by the formulas of rotation of the axes in the velocity hodograph plane by an angle β.

2. Solution of Gas-Dynamic Equations in the Flow Domain between the Source and the Shock Wave. Let us consider a spherically symmetric hypersonic flow from a source with an effective radius R_* with the following gas-dynamic parameters specified on the source surface: Mach number M_*, density ρ_*, radial velocity U_*, pressure p_*, and enthalpy of the gas h_* at an arbitrary distance r from the source center. Then, the known solution of gas-dynamic equations can be presented as [1, 2]

$$\Big(\frac{R_*}{r}\Big)^2 = \frac{\mathrm{M}}{\mathrm{M}_*}\,\Lambda^{(\gamma+1)/(2(\gamma-1))}, \qquad \frac{U}{U_*} = \frac{\mathrm{M}}{\mathrm{M}_*}\sqrt{\Lambda}, \qquad \frac{\rho}{\rho_*} = \Lambda^{1/(\gamma-1)},$$

$$\frac{p}{\rho_* U_*^2} = \frac{1}{\gamma \mathrm{M}_*^2}\,\Lambda^{\gamma/(\gamma-1)}, \qquad \frac{2h}{U_*^2} = \frac{2}{(\gamma-1)\mathrm{M}_*^2}\,\Lambda, \tag{1}$$

$$\Lambda = \frac{(\gamma-1)\mathrm{M}_*^2+2}{(\gamma-1)\mathrm{M}^2+2}, \qquad \mathrm{M} = \frac{U}{a_*}, \qquad a_* = \sqrt{\frac{\gamma p_*}{\rho_*}},$$

where γ is the ratio of specific heats of the gas and M is the local Mach number. In two limit cases, Eqs. (1) yield explicit dependences of the parameters on the distance r. In the first case, where a sonic velocity is reached on the source ($\mathrm{M}_* = 1$) and $(\gamma-1)\mathrm{M}^2 \gg 1$, we obtain

$$\mathrm{M} = R^{\gamma-1}\Big(\frac{\gamma+1}{\gamma-1}\Big)^{(\gamma+1)/4}, \qquad R = \frac{r}{R_*}, \qquad \frac{U}{U_*} = \Big(\frac{\gamma+1}{\gamma-1}\Big)^{1/2},$$

$$\frac{\rho}{\rho_*} = \Big(\frac{\gamma-1}{\gamma+1}\Big)^{1/2}\frac{1}{R^2}, \qquad \frac{p}{\rho_* U_*^2} = \frac{1}{\gamma R^{2\gamma}}\Big(\frac{\gamma-1}{\gamma+1}\Big)^{\gamma/2}. \tag{2}$$

In the other limit case, where a hypersonic velocity is reached on the source and the inequalities $(\gamma-1)\mathrm{M}_*^2 \gg 1$ and $(\gamma-1)\mathrm{M}^2 \gg 1$ are satisfied, we have

$$\mathrm{M} = \mathrm{M}_* R^{\gamma-1}, \qquad U = U_*, \qquad \frac{\rho}{\rho_*} = \frac{1}{R^2},$$

$$\frac{p}{\rho_* U_*^2} = \frac{1}{\gamma \mathrm{M}_*^2}\,\frac{1}{R^{2\gamma}}, \qquad \frac{2h}{U_*^2} = \frac{2}{(\gamma-1)\mathrm{M}_*^2}\,\frac{1}{R^{2(\gamma-1)}}. \tag{3}$$

To determine the parameters on the SW on the side of the incoming flow (domain 1 in Fig. 1), we have to assume that $R = R_S(x)$ in Eqs. (1)–(3), where $R_S(x)$ is a function determining the SW shape.

3. Rankine–Hugoniot Relations on the Shock Wave. To solve the gas-dynamic equations in the shock layer located between the SW and the contact plane (domain 2 in Fig. 1), it is necessary to formulate the boundary conditions. The Rankine–Hugoniot relations for the mass, momentum, and energy fluxes are satisfied on the oblique shock wave [6, 11]. Using the known relations for the velocity components ahead of the SW and behind it and resolving these relations with respect to the sought functions u, v, p_2, and h_2, we obtain the following relations on the SW:

$$u = U(x)[(\rho_1/\rho_2)\cos\alpha\sin(\alpha+\varphi) - \sin\alpha\cos(\alpha+\varphi)],$$

$$v = -U(x)[(\rho_1/\rho_2)\sin\alpha\sin(\alpha+\varphi) + \cos\alpha\cos(\alpha+\varphi)]; \tag{4}$$

$$p_2 = p_1 + \rho_1 U^2(x)(1-\rho_1/\rho_2)\sin^2(\alpha+\varphi),$$

$$h_2 = h_1 + (U^2(x)/2)(1-\rho_1^2/\rho_2^2)\sin^2(\alpha+\varphi). \tag{5}$$

Here, the quantities $\rho_1(x)$, $p_1(x)$, $U(x)$, and $h_1(x)$ are found from Eqs. (3) with $\mathrm{M}_* \gg 1$, $\mathrm{M} \gg 1$, and $r = R_S(x)$.

From Eqs. (5), we can easily obtain the ratio of densities on the SW:

$$\frac{\rho_1}{\rho_2} = \frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)\mathrm{M}_S^2\sin^2(\alpha+\varphi)}. \tag{6}$$

Here, $\mathrm{M}_S = \mathrm{M}_*(r_S/R_*)^{\gamma-1} = \mathrm{M}_* R_S^{\gamma-1}$ is the Mach number of the gas flow directly ahead of the SW. The following geometric relations are also valid:

$$\frac{dx_S}{dy} = \tan\,\alpha(x), \qquad r\sin\varphi = x, \qquad r\cos\varphi = D - y. \tag{7}$$

On the contact plane, we impose the no-slip condition $v\Big|_{y=0} = 0$.

4. System of Gas-Dynamic Equations in the Shock Layer. To obtain an approximate solution of the problem, it is reasonable to write the Euler equations in the Mises variables [11, 13], i.e., pass from the x and y coordinates to new independent variables x and ψ, where ψ is the stream function determined by the equality $d\psi = \rho u x\,dy - \rho v x\,dx$.

Let us consider the case with $\mathrm{M}_* \gg 1$, $\mathrm{M} \gg 1$, $p_1 \ll \rho_1 U^2$, and $h_1 \ll U^2/2$. Taking into account the boundary conditions (3)–(5) and following [1, 2, 14], we introduce the following dimensionless quantities for the shock layer (domain 2 in Fig. 1):

$$\bar{x} = \frac{x}{D}, \qquad \bar{y} = \frac{y}{D}, \qquad \bar{u} = \frac{u}{U_*}, \qquad \bar{v} = \frac{v}{U_*}, \qquad \bar{p} = \frac{p}{\rho_* U_*^2 (R_*/D)^2},$$

$$\bar{h} = \frac{2h}{U_*^2}, \qquad \bar{\rho} = \frac{\rho}{\rho_*(R_*/D)^2}, \qquad \bar{\psi} = \frac{\psi}{\rho_* U_* R_*^2}.$$

In these variables, the system of gas-dynamic equations in the shock layer takes the form

$$\frac{\partial \bar{y}}{\partial \bar{\psi}} = \frac{1}{\bar{\rho}\bar{u}\bar{x}}, \qquad \frac{\partial \bar{y}}{\partial \bar{x}} = \frac{\bar{v}}{\bar{u}},$$

$$\bar{\rho}\Big(\bar{u}\,\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\,\frac{\partial \bar{v}}{\partial \bar{x}}\Big) = -\frac{\partial \bar{p}}{\partial \bar{x}}, \qquad \frac{\partial \bar{v}}{\partial \bar{x}} = -\bar{x}\,\frac{\partial \bar{p}}{\partial \bar{\psi}}, \tag{8}$$

$$\frac{\partial}{\partial \bar{x}}\,(\bar{h} + \bar{u}^2 + \bar{v}^2) = 0, \qquad \bar{h} = \frac{2\gamma}{\gamma - 1}\,\frac{\bar{p}}{\bar{\rho}}.$$

In view of Eqs. (3), the boundary conditions on the SW (4), (5) at $\psi = \psi_S(x)$ acquire the form

$$\bar{u}_2(\bar{x}) = [(\rho_1/\rho_2)\cos\alpha\sin(\alpha+\varphi) - \sin\alpha\cos(\alpha+\varphi)],$$

$$\bar{v}_2(\bar{x}) = -[(\rho_1/\rho_2)\sin\alpha\sin(\alpha+\varphi) + \cos\alpha\cos(\alpha+\varphi)], \tag{9}$$

$$\bar{p}_2(\bar{x}) = \sin^2(\alpha+\varphi)(1 - \rho_1/\rho_2)/\bar{R}_S^2(x), \qquad \bar{h}_2(\bar{x}) = \sin^2(\alpha+\varphi)(1 - \rho_1^2/\rho_2^2).$$

With allowance for Eqs. (3) and flow geometry, the relation for the mass flux yields the dimensionless stream function on the SW:

$$\bar{\psi}_S(\bar{x}) = \int\limits_0^{\bar{x}} \frac{\sin(\alpha+\varphi)x\,dx}{\bar{R}_S^2(x)}. \tag{10}$$

Equation (10) can be written in another, clearer and more convenient form by presenting the expressions for $\sin(\alpha+\varphi)$ and $\bar{R}_S(x)$ as

$$\sin(\alpha+\varphi) = \frac{1 + \bar{x}\bar{y}'_S - \bar{y}_S}{\sqrt{1 + \bar{y}'^2_S}\sqrt{\bar{x}^2 + (1-\bar{y}_S)^2}}, \qquad \bar{R}_S(\bar{x}) = \sqrt{\bar{x}^2 + (1-\bar{y}_S)^2}.$$

Then, we obtain

$$\bar{\psi}_S(\bar{x}) = \int\limits_0^{\bar{x}} \frac{(1 + \bar{x}\bar{y}'_S - \bar{y}_S)\bar{x}}{\sqrt{1 + \bar{y}'^2_S}\,[\bar{x}^2 + (1-\bar{y}_S)^2]^{3/2}}\,d\bar{x},$$

where $\bar{y}_S(\bar{x})$ is a function determining the SW shape.

5. Asymptotic Solution of the Problem in the Shock Layer. At $\mathrm{M}_S \sin^2(\alpha+\varphi) \gg 1$, Eq. (6) yields

$$\varepsilon = \rho_1/\rho_2 \to (\gamma-1)/(\gamma+1).$$

Thus, ε is a small parameter of the problem [11]. Using the method developed in [11], we can seek for the solution in the form of power expansions of the sought functions with respect to the small parameter ε.

As it was noted in some papers (see [6, 14, 15]), however, to calculate the SW stand-off distance from the contact plane more exactly, it is necessary to use a more exact expression for the tangential component of velocity, calculating it in the next approximation in terms of ε. Therefore, we seek for the solution in the form of the following expansions (the bar above the functions is omitted):

$$\begin{gathered} y = \varepsilon y_0, \qquad u^2 = u_0^2 + \varepsilon u_f^2 + \ldots, \qquad v = \varepsilon v_0 + \ldots, \\ p = p_0 + \varepsilon p_f + \ldots, \qquad \rho = \rho_0/\varepsilon + \rho_f + \ldots, \qquad h = h_0 + \varepsilon h_f + \ldots\ . \end{gathered} \tag{11}$$

Substituting Eq. (11) into system (8), we obtain the following system of equations for the first terms of the expansions:

$$\begin{gathered} \frac{\partial u_0}{\partial x} = 0, \qquad \frac{\partial p_0}{\partial \psi} = 0, \qquad v_0 = u_0 \frac{\partial y_0}{\partial x}, \\ \frac{\partial h_0}{\partial x} = 0, \qquad h_0 = \frac{2\gamma}{\gamma+1}\frac{p_0}{\rho_0}, \qquad \rho_0 u_f \frac{\partial u_f}{\partial x} = -\frac{\partial p_0}{\partial x}, \qquad \frac{\partial y_0}{\partial \psi} = \frac{1}{\rho_0 x\sqrt{u_0^2 + \varepsilon u_f^2}}. \end{gathered} \tag{12}$$

Note that system (12) differs from a similar system obtained in [11]. It follows from Eq. (7) on the SW that

$$\varepsilon \frac{dy_{0S}}{dx} = \cot\,\alpha.$$

From here, we obtain $\alpha = \pi/2$ for $\varepsilon \to 0$. This solution, where the SW coincides with the contact plane already in the zeroth approximation in terms of ε, was used in [14]. Below, we construct a solution that takes into account the non-zero value of SW curvature already in the first approximation, in contrast to [14]. Let us demonstrate that the SW stand-off distance obtained from the solution with a correction for SW curvature is in better agreement with the results of numerical calculations than the SW stand-off distance obtained in [14] from the solution with ignored SW curvature. For this purpose, we omit the terms $O(\varepsilon)$ in the boundary conditions (9), making no assumptions about the angle α for the moment. Then, the boundary conditions (9) on the SW can be written in the form

$$\begin{gathered} \psi = \psi_S(x)\colon \qquad u_{0S}(x) = -\sin\alpha(x)\cos[\alpha(x)+\varphi(x)], \qquad u_{fS}(x) = 0, \\ v_{0S}(x) = -\cos\alpha(x)\cos[\alpha(x)+\varphi(x)], \qquad p_{0S}(x) = \sin^2[\alpha(x)+\varphi(x)]/R_S^2(x), \\ h_{0S}(x) = \sin^2[\alpha(x)+\varphi(x)]. \end{gathered} \tag{13}$$

With allowance for Eqs. (13), the solution of system (12) acquires the following form with accuracy to an arbitrary angle α:

$$\begin{gathered} u_0(\psi) = -\sin\alpha(t)\cos[\alpha(t)+\varphi(t)], \qquad p_0(x) = \sin^2[(\alpha(x)+\varphi(x)]/R_S^2(x), \\ h_0(t) = \sin^2[\alpha(t)+\varphi(t)], \qquad \rho_0(x,t) = (2\gamma/(\gamma+1))p_0(x)/h_0(t) \end{gathered} \tag{14}$$

(t is the coordinate of the point where the streamline ψ enters the shock layer). The correction to the tangential component of velocity has the form

$$u_f^2(x,t) = -\frac{\gamma+1}{\gamma} h_0(t) \ln\Big(\frac{p_0(x)}{p_0(t)}\Big), \tag{15}$$

and $u^2(x,t) = u_0^2(t) + \varepsilon u_f^2(x,t)$. The current value of ψ corresponding to the coordinate t is determined by the formula

$$d\psi = \frac{\sin[\alpha(t)+\varphi(t)]t}{R_S^2(t)}\,dt.$$

The coordinate $y_0(x,t)$ is found from Eqs. (8) and the expressions obtained above:

$$y_0(x,t) = \frac{\gamma+1}{2\gamma}\,\frac{1}{xp_0(x)}\int\limits_0^t \frac{zp_0(z)\sin[\alpha(z)+\varphi(z)]}{u(x,z)}\,dz. \qquad (16)$$

Substituting Eqs. (14) and (15) into Eq. (16), we obtain the relation for the function determining the SW shape at $t = x$:

$$y_0^{\rm sh}(x) = \frac{\gamma+1}{2\gamma}\,\frac{1}{xp_0(x)}\int\limits_0^x \frac{zp_0(z)\sin[\alpha(z)+\varphi(z)]}{u(x,z)}\,dz, \qquad (17)$$

where

$$p_0(x) = \frac{(1-y_S+xy'_S)^2}{(1+y'^2_S)[x^2+(1-y_S)^2]^2}, \qquad y'_S = \frac{dy_S}{dx},$$

$$u(x,z) = \sqrt{\sin^2\alpha(z)\cos^2[\alpha(z)+\varphi(z)] - b_1h_0(z)\ln(p_0(x)/p_0(z))}, \quad b_1 = (\gamma-1)/\gamma, \qquad y_S(x) = \varepsilon y_0^{\rm sh}(x).$$

Solution (17) obtained from the continuity equation has to be compatible with the geometric relation (7) for the angle of inclination $\alpha(x)$ in the vicinity of the axis of symmetry. In a particular case with $\alpha = \pi/2$, Eq. (17) yields the formula determining the SW shape and the limiting expressions for u_0, p_0, and h_0 [14]. In view of Eq. (7), however, Eq. (17) is an integrodifferential equation, because there are functions of $\alpha(x)$ in the integral. The function $\alpha(x)$, in turn, depends on the derivative y'_{Sx}. To construct the solution of the integral equation (17) and refine the value of the SW stand-off distance from the contact plane, we expand the functions involved in Eq. (17) into a series with respect to the powers of x, confining ourselves to terms of the order $O(x^2)$. As a result, we obtain algebraic equations for determining the curvature $K_{S0} = R_{S0}^{-1}$ and the SW stand-off distance $y_S^{\rm sh}(0) = \Delta_0$. Nondimensionalizing Eq. (7), substituting $\alpha = \pi/2 - K_{S0}x + \ldots$ (K_{S0} is the SW curvature at the critical point) into this equation, expanding the function $\tan(K_{S0}x)$ into a series, and integrating the resultant expression, we obtain

$$y_S^{\rm sh} = \Delta_0 + \frac{K_SD}{\varepsilon}\,\frac{x^2}{2}, \qquad y_S = \varepsilon\Delta_0 + \frac{K_SD}{2}\,x^2. \qquad (18)$$

We introduce the notation $K_SD = K_0$. Then, omitting the terms of the order $O(\varepsilon^2)$, $O(x^4)$, and also $O(K_0^2)$, we obtain the expansions

$$R_S^2(x) = 1 + x^2(1-K_0), \qquad \sin(\alpha+\varphi) = \frac{1+K_0x^2/2}{1+x^2(1-K_0)/2},$$

$$\cot^2(\alpha+\varphi) = (K_0-1)^2x^2 + \ldots, \qquad (19)$$

$$\frac{R_S^2(x)}{\sin^2(\alpha+\varphi)} = 1 + Ax^2 + \ldots, \qquad A = 2 - 3K_0, \qquad \ln\Big(\frac{p_0(x)}{p_0(t)}\Big) = A(t^2 - x^2) + \ldots\ .$$

Substituting expansions (19) into the right side of Eq. (17) and calculating the integral, we have

$$y_0^{\rm sh}(x) = \frac{1}{1+\varepsilon}\,\frac{1}{k_0+m_0}\Big(1 + \frac{Ax^2}{3}\,\frac{m_0+2k_0}{m_0+k_0}\Big), \qquad (20)$$

where $k_0 = \sqrt{B + b_1A}$, $m_0 = \sqrt{b_1A}$, and $B = K_0 - 1 - b_1A$. The left side of Eq. (20) is expansion (18). Comparing the coefficients at the powers x^0 and x^2 in the left and right sides of Eq. (20) and applying some transformations, we obtain two algebraic equations for Δ_0 and K_0:

$$y_S(0) = \varepsilon\Delta_0 = \frac{b}{4}\,\frac{1}{1-K_0+\sqrt{b(2-3K_0)/2}}, \qquad b = \frac{4\varepsilon}{1+\varepsilon} = 2\,\frac{\gamma-1}{\gamma}; \qquad (21)$$

$$K_0 = \frac{b}{6}\,\frac{2-3K_0}{[1-K_0+\sqrt{b(2-3K_0)/2}\,]^2}\,[\sqrt{b(2-3K_0)/2} + 2(1-K_0)]. \qquad (22)$$

Equation (22) does not involve Δ_0 and can be solved independent of Eq. (21).

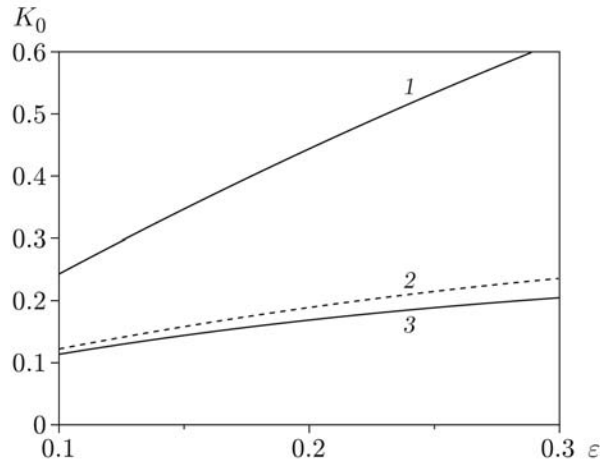

Fig. 2

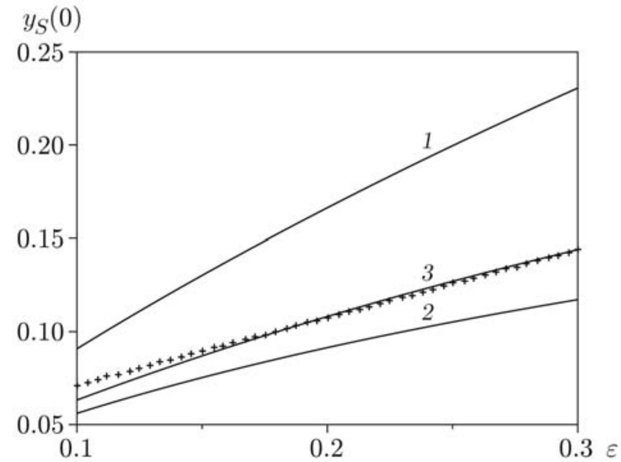

Fig. 3

Fig. 2. SW curvature K_0 versus ε: calculation with no allowance for the correction εu_f^2 [14] (curve 1), calculation with allowance for the correction εu_f^2 (curve 2), and calculation by Eq. (23) (curve 3).

Fig. 3. SW stand-off distance $y_S(0)$ versus ε: calculation with no allowance for the correction εu_f^2 [14] (curve 1), calculation with allowance for the correction εu_f^2 (curve 2), and calculation by Eqs. (21) and (22) (curve 3); the points are the results of the numerical calculation [2].

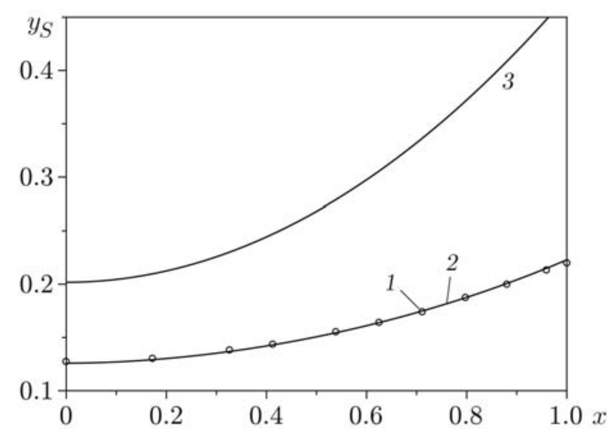

Fig. 4

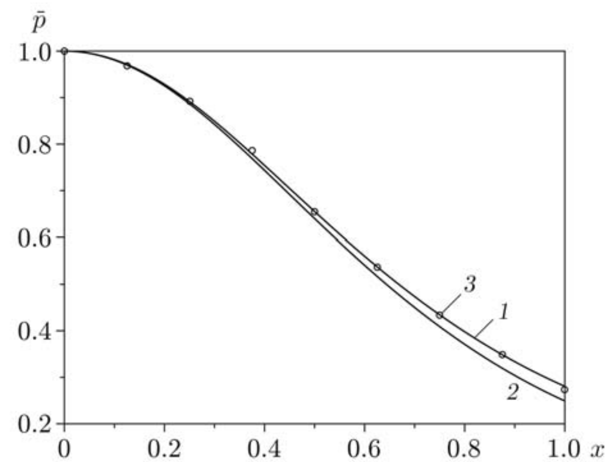

Fig. 5

Fig. 4. Calculated SW shape ($\gamma = 1.67$): results of the numerical calculation [2] (points 1), calculation by Eq. (24) (curve 2), and calculation with no allowance for the correction εu_f^2 [14] (curve 3).

Fig. 5. Pressure distribution on the contact plane at $\gamma = 1.4$: calculation by Eq. (25) (curve 1), calculation by the Newton formula (curve 2), and results of the numerical calculation [2] (points 3).

Equations (22) was solved numerically, but it can also be solved approximately in an explicit form by presenting the right side in a linear form with respect to K_0. Then, in the linear approximation, we obtain the solution

$$K_0 = \frac{b(2+\sqrt{b}\,)(1+\sqrt{b}\,)/3}{(1+\sqrt{b}\,)^3 + b(1+3b/4+9\sqrt{b}/4)/3}. \tag{23}$$

Calculations by Eq. (23) yield the values of K_0 that differ only in the third digit from the exact solutions obtained by the numerical solution of Eq. (22).

6. Calculation Results. Figures 2 and 3 show the SW curvature K_0 and SW stand-off distance $y_S(0)$ as functions of ε. It is seen that the use of the correction εu_f^2 to the tangential velocity exerts a considerable effect on the values of K_0 and $y_S(0)$. The results calculated by Eqs. (21), (22) (curve 3 in Fig. 3) are in better agreement [as compared with the results calculated with allowance for the correction (curve 2)] with the numerical solution [2] (points); the difference is smaller than 10%.

With allowance for the found values of $y_S(0)$ and K_0, the equation determining the SW shape near the axis of symmetry acquires the form similar to Eq. (18):

$$y_S(x) = y_S(0) + K_0 x^2/2. \tag{24}$$

Figure 4 shows the SW shape calculated at $\gamma = 1.67$. It is seen that curves 1 and 2 are in good agreement in the entire interval $0 < x < 1$. At the same time, curve 3 differ substantially from curves 1 and 2. With allowance for the expansions obtained above and the found values of $y_S(0)$ and K_0, the expression for the pressure in the shock layer takes the form

$$p_0(x) = \frac{[1 + K_0 x^2/2 - y_0(x)]^2}{[1 + x^2(1 - K_0) - 2y_0(0)]^2}. \tag{25}$$

Figure 5 shows the pressure distributions $\bar{p}(x) = p_0(x)/p_0(0)$ on the contact plane at $\gamma = 1.4$. It is seen that curves 1 and 3 almost coincide. The calculations show that the parameter γ exerts a minor effect on the pressure distribution.

Conclusions. Thus, it is demonstrated that the solution obtained [SW stand-off distance $y_S(0)$, SW shape, and pressure distribution $\bar{p}(x)$] with allowance for the correction for the SW curvature agrees well with the numerical solution [2]. It is found that the allowance for the correction to the tangential component of velocity exerts a significant effect on the SW stand-off distance and shape.

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